

Trial Higher School Certificate Examination
2009

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen
- Begin each question on a new booklet
- Write your student number on each page
- All necessary working must be shown.
- Diagrams are not to scale.
- Board-approved calculators may be used.
- The mark allocated for each question is listed at the side of the question.

2 questions per booklet.

Total Marks - 120

- Attempt ALL questions.
- All questions are of equal value.

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 - (12 marks)

- a) Factorise $4 - 4p^3$

2

b) Solve $\frac{x}{2} - \frac{1-5x}{4} = x$

2

c) Express $\frac{\sqrt{2}+1}{1-\sqrt{2}}$ in the form $a + b\sqrt{2}$ where a and b are rational numbers.

2

- d) Graph the solution of $|2x - 1| < 7$ on the number line.

2

e) Evaluate $\frac{9.7 \times 4.16 - (3.62)^2}{\sqrt{4.51}}$ correct to 2 decimal places.

1

f) Given that $f(p) = p^2 + p$, find the values of m for which $f''(m) = f(m)$

3

Question 2 - (12 marks)

The vertices of a triangle are $A(-1, 0)$ and $B(2, 6)$ and $C(x, y)$. Given that C lies on the x axis, to the right of A such that $\hat{BAC} = \hat{BCA} = \theta$.

- | | |
|---|---|
| a) Mark this information on a number plane. | 2 |
| b) Find the gradient of AB and hence find θ , to the nearest degree. | 2 |
| c) Show that the equation of the line AB is $y = 2x + 2$ | 1 |
| d) Explain why BC has a gradient of -2 | 1 |
| e) If D is the point $(-2, 5)$ on this diagram, find the perpendicular distance of D from the line AB . | 3 |
| f) Hence, calculate the area of the quadrilateral $ADBC$. | 3 |

Question 3 - (12 marks)

a) Differentiate

(i) $x^3 e^{3x}$

(ii) $\frac{x}{\sin x}$

b) Find

(i) $\int \cos \frac{3x}{2} dx$

(ii) $\int \sqrt{e^x} dx$

c) Find the equation of the normal to the curve $y = \sqrt{x+5}$ at the point where $x = 4$.

2

2

2

2

4

Question 4 - (12 marks)

- a) Sketch the graph of $y = 2 + \cos 2x$ for $0 \leq x < 2\pi$

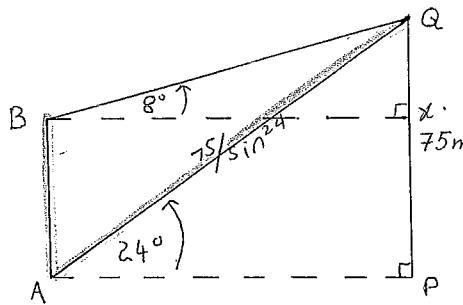
Marks

3

- b) Show that $(1 - \cos^2\theta) \cot^2\theta = \cos^2\theta$

2

- c) Two towers AB and PQ (as shown) stand on level ground. The angles of elevation of the top of the taller tower from the top and the bottom of the shorter tower are 8° and 24° respectively. The height of the taller tower is 75 metres.



- (i) Explain why $A\hat{Q}B = 16^\circ$

1

- (ii) Show that $AB = \frac{AQ \sin 16^\circ}{\sin 98^\circ}$

2

- (iii) Show that $AQ = \frac{75}{\sin 24^\circ}$

2

- (iv) Hence find the height of the shorter tower correct to the nearest metre.

2

Question 5 - (12 marks)

- a) Solve $m^4 + 12m^2 - 64 = 0$

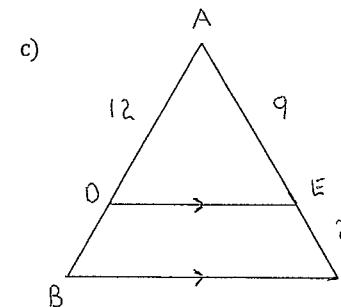
3

- b) The line $3x + 5y = k$ is a tangent to $y = x^2 - x - 1$.

4

* (i) Explain why the discriminant of $3x + 5(x^2 - x - 1) = k$ must equal zero.

* (ii) Hence, find the value of k .



The diagram shows ΔABC
 $DE \parallel BC$ and $AD = 12$
 $AE = 9$ and $EC = 2$

2

Find the length of DB giving reasons.

- d) Solve $2\sin^2 x + \cos x = 2$ $0^\circ \leq x \leq 2\pi$

3

Question 6 - (12 marks)

a) For the parabola $(x - 2)^2 = 8y + 12$ find the following:

(i) the vertex

1

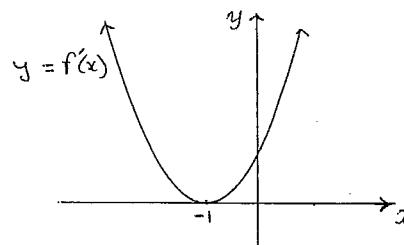
(ii) the focus

2

(iii) the equation of the directrix

1

b) Consider the function $y = f(x)$, whose gradient function is shown as a parabola in the sketch below.



(i) Comment on the sign of the gradient function for all $x \neq -1$

1

(ii) What can you conclude about $y = f(x)$ when $x = -1$

1

(iii) Sketch a graph for $y = f''(x)$

1

(iv) Sketch a possible graph for $y = f(x)$

2

c) Show that $\int_2^3 \frac{3x}{x^2 - 3} dx = \log \sqrt{216}$

3

Question 7 - (12 marks)

a) Solve $x^2 - x - 12 > 0$

2

b) (i) For what values of k does the series given have a limiting sum.

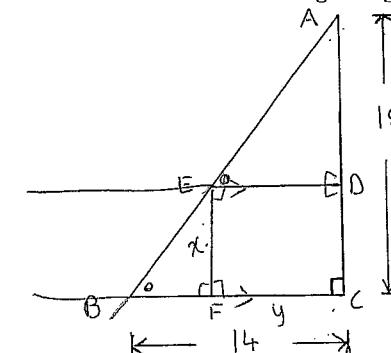
$$18 + 24k + 32k^2 + \dots$$

2

(ii) If $k = \frac{1}{6}$ find the limiting sum.

1

c) A rectangle $EDCF$ is constructed inside a right-angled triangle ABC as shown in diagram.



(i) If $EF = x$ cm and $FC = y$ cm, show that

$$\frac{14-y}{x} = \frac{14}{18}$$

2

(ii) Show that the area of the rectangle is given by $\frac{9}{7}(14y - y^2)$

2

* (iii) Hence show that the rectangle with the greatest area has an area equal to half the area of the triangle.

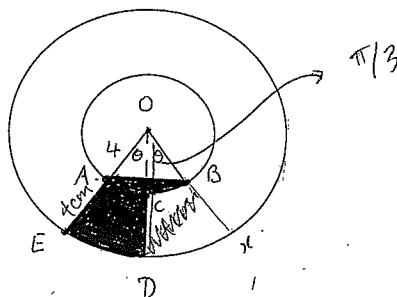
3

Question 8 – (12 marks)

Marks

- a) Jane creates a design piece for a puzzle she is making from two concentric circles as shown.

4



If $\theta = \frac{\pi}{3}$ radians and $OA = 4 \text{ cm}$ and $AE = 4 \text{ cm}$.

Find the area of the shaded piece she has created, correct to the nearest cm^2 .

- b) At the beginning of each year a woman invests \$1400 in a superannuation fund, on which she is paid 7.8% p.a. interest compounded yearly. Find:

(i) the amount of interest earned in the first year.

2

* (ii) the total her investment amounts to after 30 years.

3

- c) Use Simpson's Rule with 5 function values to find the approximate value of

$$\int_2^6 \log x \, dx$$

correct to 1 decimal place.

3

Question 9 – (12 marks)

Marks

- a) Consider the curve given by $y = 1 - 3x + x^3$ for the domain $-2 \leq x \leq 3$

(i) Find the stationary points and determine their nature.

2

(ii) Find the point of inflexion.

1

(iii) Sketch the curve for $-2 \leq x \leq 3$

2

(iv) What is the global maximum of the function for the domain $-2 \leq x \leq 3$

1

- b) (i) Differentiate $(x^2 - 2x)e^x$

2

(ii) Hence evaluate $\int_0^1 x^2 e^x \, dx$

2

- c) Given $y = 3 \sin 2x$ write down

2

(i) the period.

(ii) the amplitude of this graph.

Question 10 – (12 marks)

Marks

- a) (i) Sketch the graph of $y = \sec x$ $0 < x < 2\pi$ 2
- (ii) The curve $y = \sec x$ is rotated around the x -axis between $x = 0$ and $x = \frac{\pi}{3}$ to form a solid. Find the volume of this solid. 3
- b) A particle moves in such a way that its distance, x metres, from the origin after t seconds is given by $x = 3 + 18t - t^3$ for $t \geq 0$
- (i) Find an equation for its velocity after t seconds. 1
- (ii) At what time does the particle stop? 1
- (iii) Where is the particle initially? 1
- (iv) Find the velocity after 1 second. 1
- (v) How far has the particle travelled in the first second. 1
- (vi) What is the particle's acceleration after 2 seconds. 2

Question 1

$$(a) 4 - 4p^3 = 4(1 - p^3) \\ = 4(1-p)(1+p+p^2)$$

$$(b) \frac{x}{2} - \frac{1-5x}{4} = x$$

$$\textcircled{4} \quad 2x - (1-5x) = 4x \\ 2x - 1 + 5x - 4x = 0 \\ 3x = 1 \\ x = \frac{1}{3}$$

$$(c) \frac{\sqrt{2}+1}{1-\sqrt{2}} = \frac{(\sqrt{2}+1)(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} \\ = \frac{2+2\sqrt{2}+1}{1-2} \\ = \frac{3+2\sqrt{2}}{-1} \\ = -3-2\sqrt{2}$$

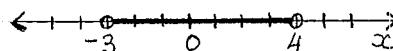
$$\therefore a = -3, b = -2$$

$$(d) |2x-1| < 7$$

$$-7 < 2x-1 < 7$$

$$-6 < 2x < 8$$

$$\textcircled{1} \quad -3 < x < 4$$



$$(e) \frac{9.7 \times 4.16 - (3 \cdot 6.2)^2}{\sqrt{451}} \\ = 12.83039374... \\ \approx 12.83$$

$$(f) f(p) = p^2 + p$$

$$f(m) = m^2 + m$$

$$f'(m) = 2m+1 \quad f''(m) = 2$$

$$m^2 + m = 2$$

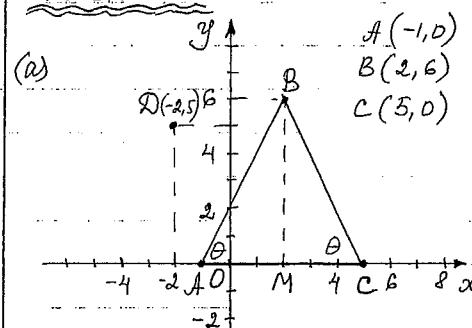
$$m^2 + m - 2 = 0$$

$$(m-1)(m+2) = 0$$

$$m=1 \text{ or } m=-2$$

Question 2

(a)



$$(b) \text{grad. AB} = \frac{6-0}{2+1} = \frac{6}{3} = 2$$

$$\tan \theta = 2 \quad \therefore \theta = \tan^{-1}(2) = 63^\circ$$

$$(c) \text{grad. AB} = 2 \text{ and } A = (-1, 0) \\ \text{using point-gradient:}$$

$$y - 0 = 2(x + 1)$$

$$y = 2x + 2, \text{ as required.}$$

$$(d) \text{grad. BC} = \tan(180 - \theta)$$

$$= -\tan \theta$$

$$= -2$$

(e) Line AB in general form:

$$2x - y + 2 = 0$$

$$D = (-2, 5)$$

$$\text{Perpendicular distance} \\ d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\therefore d = \frac{|2 \times (-2) - 5 + 2|}{\sqrt{2^2 + (-1)^2}}$$

$$= \frac{|-7|}{\sqrt{5}} = \frac{7}{\sqrt{5}} \text{ units}$$

$$(f) A_{ADBC} = A_{ADB} + A_{ABC}$$

$$= \frac{1}{2} AB \times d + \frac{1}{2} AC \times BM$$

$$AB = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$$

$$AC = 6 ; BM = 6$$

$$A_{ADB} = \frac{1}{2} \times \frac{3\sqrt{5} \times 7}{\sqrt{5}} + \frac{1}{2} \times 6 \times 6$$

$$= \frac{21}{2} + 18 = 28.5 \text{ units}^2$$

$$(c) y = \sqrt{x+5}, \text{ normal at } x=4$$

$$y = (x+5)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x+5)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x+5}}$$

$$\text{at } x=4 \quad \frac{dy}{dx} = \frac{1}{2 \times 3} = \frac{1}{6}$$

$$\therefore \text{the grad. of the normal} = -6$$

$$\text{when } x=4 \quad y = \sqrt{9} = 3$$

$$\therefore \text{the point is } (4, 3)$$

$$y = 3 = -6(x-4)$$

$$y = -6x + 24 + 3$$

The normal is:

$$6x + y - 24 = 0$$

$$\textcircled{1} \quad y = -6x + 27$$

Question 3

$$(a) (i) \text{let } y = x^3 e^{3x}$$

$$\text{let } u = x^3 \text{ and } v = e^{3x}$$

$$\frac{du}{dx} = 3x^2 \text{ and } \frac{dv}{dx} = 3e^{3x}$$

$$\frac{dy}{dx} = e^{3x} \times 3x^2 + x^3 \times 3e^{3x} \\ = 3x^2 e^{3x} (1+x)$$

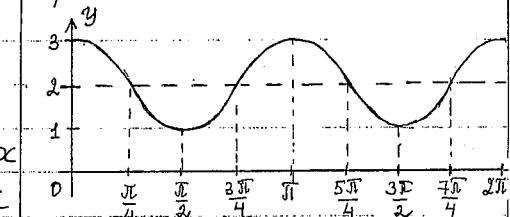
$$(ii) \text{let } y = \frac{x}{\sin x}$$

$$\text{let } u = x \text{ and } v = \sin x$$

$$\frac{du}{dx} = 1 \text{ and } \frac{dv}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{\sin x - x \cos x}{\sin^2 x}$$

$$(a) y = 2 + \cos 2x \text{ for } 0 \leq x \leq 2\pi$$

 period = π


$$(b) (1 - \cos^2 \theta) \cot^2 \theta = \cos^2 \theta$$

$$\text{LHS} = (1 - \cos^2 \theta) \cot^2 \theta$$

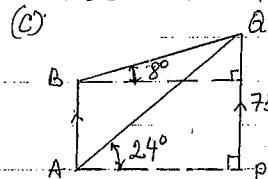
$$= \sin^2 \theta \times \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= \cos^2 \theta \\ = \text{RHS as required.}$$

$$(b) \int \cos \frac{3x}{2} dx = \frac{2}{3} \sin \frac{3x}{2} + C$$

$$(ii) \int \sqrt{e^x} dx = \int e^{\frac{x}{2}} dx \\ = 2 e^{\frac{x}{2}} + C \text{ or} \\ = 2 \sqrt{e^x} + C$$

Question 4 continued



(i) $\widehat{AQP} = 180^\circ - 90^\circ - 24^\circ = 66^\circ$
(angle sum of $\triangle APQ$ is 180°)

$\widehat{QAB} = \widehat{AQP} = 66^\circ$
(alternate angles, $AB \parallel PQ$)

$\widehat{ABQ} = 90^\circ + 8^\circ = 98^\circ$
 $\widehat{AQB} = 180^\circ - 66^\circ - 98^\circ = 16^\circ$

(angle sum of $\triangle ABQ$ is 180°)

(ii) In $\triangle A B Q$

$$\frac{AB}{\sin 16^\circ} = \frac{AQ}{\sin 98^\circ} \quad (\text{sine rule})$$

$$\therefore AB = \frac{AQ \sin 16^\circ}{\sin 98^\circ}$$

(iii) In $\triangle APQ$ $\sin 24^\circ = \frac{45}{AQ}$

$$\therefore AQ = \frac{45}{\sin 24^\circ}$$

(iv) $AB = \frac{45}{\sin 24^\circ} \times \frac{\sin 16^\circ}{\sin 98^\circ}$
= $51.32550878\dots$
 ≈ 51 metre.

\therefore The height of the lower tower is 51 metres.

Question 5

(a) $m^4 + 12m^2 - 64 = 0$

$$(m^2 + 16)(m^2 - 4) = 0$$

$$m^2 + 16 = 0 \quad \text{or} \quad m^2 - 4 = 0$$

$$m^2 = -16 \quad \text{or} \quad m^2 = 4$$

(no solutions)

(b) Line: $3x + 5y = k$ ①

Parabola: $y = x^2 - x - 1$ ②

In order to find points of intersection substitute ② into ①:

$$3x + 5(x^2 - x - 1) = k \quad ③$$

If the line is a tangent to the parabola (one common point) ③ must have only one solution (one double root) and this happens when the discriminant $\Delta = 0$.

(ii)

$$3x + 5x^2 - 5x - 5 - k = 0$$

$$5x^2 - 2x - (5+k) = 0$$

$$a = 5, b = -2, c = -(5+k)$$

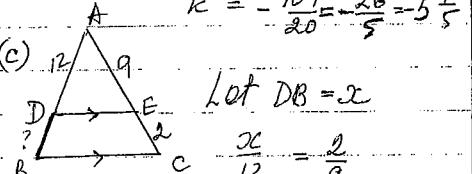
$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= 4 - 4 \times 5 \times(-(5+k)) \\ &= 4 + 100 + 20k \\ &= 20k + 104\end{aligned}$$

Since $\Delta = 0$

$$20k + 104 = 0$$

$$20k = -104$$

$$k = -\frac{104}{20} = -\frac{26}{5} = -5\frac{1}{5}$$



(intercepts cut by parallel lines are in the same ratio)

$$\therefore \frac{AD}{DB} = \frac{2}{12} = \frac{2}{9} = \frac{8}{36}$$

$$\therefore \frac{AE}{EC} = \frac{2}{9} = \frac{2}{3}$$

$$\therefore DB = \frac{8}{3} = 2\frac{2}{3}$$

Question 6 continued

(d)

$$0 \leq x \leq 2\pi$$

$$2\sin^2 x + \cos x = 2$$

$$2(1 - \cos^2 x) + \cos x - 2 = 0$$

$$2 - 2\cos^2 x + \cos x - 2 = 0$$

$$-2\cos^2 x + \cos x = 0$$

Let $\cos x = u$, then:

$$-2u^2 + u = 0$$

$$u(1 - 2u) = 0$$

$$u = 0 \quad \text{or} \quad 1 - 2u = 0$$

$$2u = 1$$

$$u = \frac{1}{2}$$

$$\cos x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Question 6

(i)

$$(x-2)^2 = 8y + 12$$

$$(x-2)^2 = 8(y + \frac{3}{2})$$

$$\text{Vertex} = (2, -\frac{3}{2})$$

(ii) $4a = 8$

$\therefore a = 2$ (focal length)

Since the parabola is concave up the focus is $(2, -\frac{3}{2} + 2)$
 $= (2, \frac{1}{2})$

(iii) $y = -\frac{3}{2} - 2 = -\frac{7}{2} = -3\frac{1}{2}$

\therefore the directrix is

$$y = -3\frac{1}{2}$$

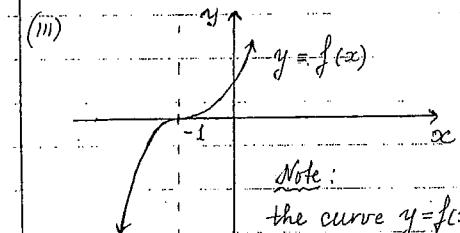
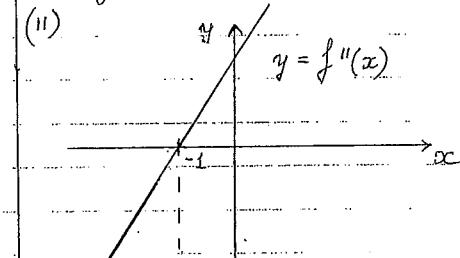
(b) (i) For all $x \neq -1$ the

gradient function is positive.

(ii) at $x = -1$ the gradient

function is zero,

$\therefore y = f(x)$ has a stationary point at $x = -1$.



Note:
the curve $y = f(x)$ can be moved up or down, but needs

to have a horizontal point of inflection at $x = -1$.

(c)

$$\int_{-2}^3 \frac{3x}{x^2 - 3} dx = \int_{-2}^3 \frac{\frac{3}{2} \times 2x}{x^2 - 3} dx$$

$$= \frac{3}{2} \int_{-2}^3 \frac{2x}{x^2 - 3} dx$$

$$= \frac{3}{2} \left[\log(x^2 - 3) \right]_{-2}^3$$

$$= \frac{3}{2} \left(\log 6 - \log 1 \right)$$

$$= \frac{3}{2} \log 6$$

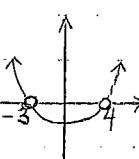
$$= \log 6^{\frac{3}{2}} = \log \sqrt{216}$$

Question 7

$$(a) x^2 - x - 12 > 0 \\ (x-4)(x+3) > 0$$

From the graph:

$$x < -3 \text{ or } x > 4$$



$$\frac{BF}{EF} = \frac{BC}{AC} \quad (\text{corresponding sides in similar triangles are in the same ratio})$$

$$\therefore \frac{14-y}{x} = \frac{14}{18} \quad \text{as required}$$

$$(b) 18 + 24k + 32k^2 + \dots$$

$$(i) r = \frac{24k}{18} = \frac{4k}{3}$$

For limiting sum to exist:

$$-1 < r < 1$$

$$-1 < \frac{4k}{3} < 1$$

$$-3 < 4k < 3$$

$$-\frac{3}{4} < k < \frac{3}{4}$$

$$(ii) \text{ When } k = \frac{1}{6}$$

$$r = \frac{4}{3} \times \frac{1}{6} = \frac{4}{18} = \frac{2}{9}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{18}{1 - \frac{2}{9}} = \frac{18}{\frac{7}{9}} = \frac{18 \times 9}{7}$$

$$S_{\infty} = \frac{162}{7}$$

$$(c) (i) \text{ In}$$

 $\triangle EBF$ and $\triangle ABC$

$$\angle EBF = \angle ABC \quad (\text{common})$$

Since $FE \parallel CA$

$$(\text{EDCF is a rectangle})$$

$$\angle BEF = \angle BAC$$

(corresponding angles on parallel lines)

 $\therefore \triangle EBF \sim \triangle ABC$ (equilateral)

$$(ii) \text{ From part (i):}$$

$$x = \frac{18(14-y)}{14}$$

$$= \frac{9}{7}(14-y)$$

$$A_{\#} = xy$$

$$= \frac{9}{7}(14-y) \times y$$

$$= \frac{9}{7}(14y - y^2)$$

as required.

$$(iii) A = \frac{9}{7}(14y - y^2) = 18y - \frac{9y^2}{7}$$

Stationary points when $\frac{dA}{dy} = 0$.

$$\frac{dA}{dy} = 18 - \frac{18y}{7}$$

$$18 - \frac{18y}{7} = 0$$

$$18y = 7 \times 18$$

$$y = 7$$

$$\frac{d^2A}{dy^2} = -\frac{18}{7}$$

$$\frac{d^2A}{dy^2} < 0 \quad \therefore \text{concave down}$$

and the stationary point is the maximum value of the Area.

$$\text{when } y = 7$$

$$x = \frac{9}{7}(14-7) = 9$$

$$A_{\square} = 7 \times 9 = 63; A_{\Delta} = \frac{1}{2} \times 14 \times 18 = 7 \times 18 = 126$$

$\therefore \max A_{\square} = \frac{1}{2} A_{\Delta}$

Question 8

$$(a) A = A_{\text{small seg.}} + (A_{\text{large sect.}} - A_{\text{small seg.}})$$

$$= \frac{1}{2} r^2(2\theta - \sin 2\theta) + \left(\frac{1}{2} R^2\theta - \frac{1}{2} r^2\theta \right)$$

$$= \frac{1}{2} 4^2 \left(\frac{2\pi}{3} - \sin \frac{2\pi}{3} \right) + \left(\frac{1}{2} 8^2 \times \frac{\pi}{3} - \frac{1}{2} 4^2 \times \frac{\pi}{3} \right)$$

$$= 8 \left(\frac{2\pi}{3} - \sin \frac{2\pi}{3} \right) + \frac{32\pi}{3} - \frac{8\pi}{3}$$

$$= 8 \left(\frac{2\pi}{3} - \sin \frac{2\pi}{3} \right) + \frac{24\pi}{3}$$

$$= 8 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) + \frac{32\pi}{3} - 24\sqrt{3} + 4\pi$$

$$= \frac{80\pi}{3} - 24\sqrt{3}$$

$$= 34.989698, \dots$$

$$\approx 35 \text{ cm}^2$$

 \therefore The area of the shaded piece is 35 cm^2

(b)

(i) Interest after 1 year =

$$= 1400 \times 1.078^1 - 1400$$

$$= \$ 109.20$$

(ii)

1st inst. amounts to 1400×1.078^{30} 2nd inst. amounts to 1400×1.078^{29} 3rd inst. amounts to 1400×1.078^1 This is a GP with $a = 1400 \times 1.078^1$

$$r = 1.078 \text{ and}$$

30 terms.

We can use the sum of a GP formula:

$$\text{Total investment} = \frac{1400 \times 1.078 (1.078^{30} - 1)}{0.078}$$

$$\approx 164,819.64$$

(to the nearest cent)

$$\approx 164,820$$

(to the nearest \$)

(c)

x	2	3	4	5	6
$\log x$	$\log 2$	$\log 3$	$\log 4$	$\log 5$	$\log 6$

$$h = \frac{1}{2}$$

$$\int \log x \, dx =$$

$$= \frac{1}{3} (\log 2 + 4 \log 3 + \log 4) + \frac{1}{3} (\log 4 + 4 \log 5 + \log 6)$$

$$= \frac{1}{3} (\log 2 + 4 (\log 3 + \log 5) + 2 \log 4 + \log 6)$$

$$= 5.36323 \dots$$

$$\approx 5.4 \text{ (to 1 d.p.)}$$

Question 9

$$(a) y = 1 - 3x + x^3; -2 \leq x \leq 3$$

$$(i) \text{ Stationary points } \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 3x^2 - 3$$

$$3x^2 - 3 = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$x = 1 \text{ or } x = -1$$

$$\text{when } x = 1 \quad \frac{d^2y}{dx^2} = 6x = 6 > 0 \text{ (min)}$$

$$y = -1$$

$$\therefore (1, -1) \text{ is a minimum turning point}$$

$$\text{when } x = -1 \quad \frac{d^2y}{dx^2} = -6 < 0 \text{ (max)}$$

$$y = 3$$

$$(-1, 3) \text{ is a maximum turning point.}$$

$$(ii) \text{ Inflection when } \frac{d^2y}{dx^2} = 0$$

$$6x = 0 \quad \text{when } x = 0$$

$$\frac{x}{-1} \mid 0 \mid 1$$

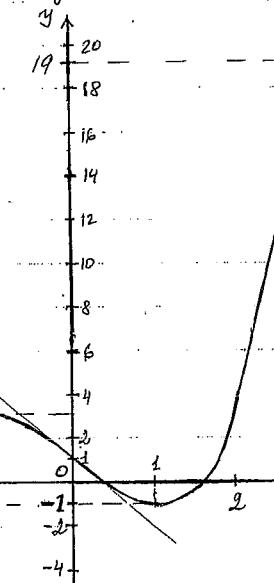
$$\frac{y''}{-1} \mid - \mid +$$

$$\frac{d^2y}{dx^2} \text{ changes sign,}$$

$$\therefore (0, 1) \text{ is a point of inflection.}$$

Q.9 continues domain $-2 \leq x \leq 3$

(a) (ii) when
 $x = -2$
 $y = 1+6-8 = -1$
when
 $x = 3$
 $y = 1-9+27 = 19$



(iv) global maximum is $y = 19$, occurs at $x = 3$.

(b)

(i) $\frac{d}{dx}(x^2-2x)e^x =$
 $= e^x(2x-2)+(x^2-2x)e^x$

$$= e^x(x^2-2)$$

$$= -2e^x$$

(ii) $\int_0^1 (x^2e^x - 2e^x) dx = [(x^2-2x)e^x]_0^1$

$$\int_0^1 x^2e^x dx - \int_0^1 2e^x dx = [(x^2-2x)e^x]_0^1$$

$$\therefore \int_0^1 x^2e^x dx = [(x^2-2x)e^x]_0^1 + \int_0^1 2e^x dx$$

$$= -e^1 + [2e^x]_0^1$$

$$= -e + (2e-2)$$

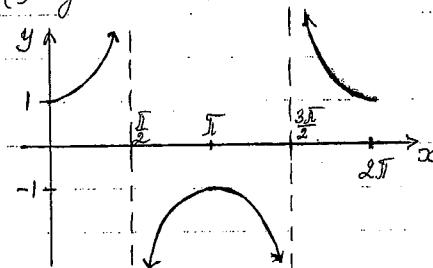
$$= e - 2$$

(c) $y = 3\sin 2x$

(i) period = $\frac{2\pi}{2} = \pi$
amplitude = 3

Question 10

(a) $y = \sec x \quad 0 < x < 2\pi$



(ii) $V = \pi \int_a^b [f(x)]^2 dx$

$$V = \pi \int_0^{\frac{\pi}{3}} \sec^2 x dx$$

$$= \pi [\tan x]_0^{\frac{\pi}{3}}$$

$$= \pi (\tan \frac{\pi}{3} - \tan 0)$$

$$= \pi \times \frac{\sqrt{3}}{1}$$

$$= \sqrt{3}\pi \text{ units}^3$$

(b) $x = 3+18t-t^3$ for $t \geq 0$

(i) $V = 18-3t^2$

(ii) $V=0 \quad 18-3t^2=0$

$$3t^2=18$$

$$t^2=6$$

$$t=\sqrt{6}$$

: the particle stops after $\sqrt{6}$ seconds.

(iii) $t=0 \quad x=3$

: the particle is 3 m. to the positive direction from the origin.

(iv) $V = 18-3 = 15 \text{ m/s}$

Question 10 continues

(b)

(v)

$$x = 3+18t-t^3$$

$$t = 1 \quad x = 3+18-1 = 20$$

distance travelled =

$$|x_1 - x_0| = 20-3 = 17 \text{ m}$$

(vi) $a = -6t$

when $t = 2$

$$a = -12 \text{ m/s}^2$$